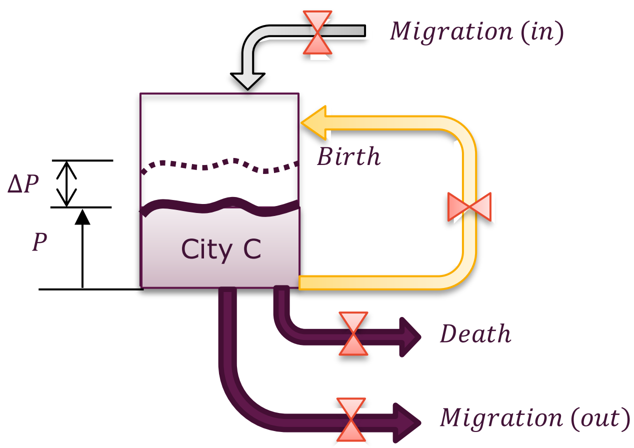
System Dynamics

What is system dynamics?

Owing its heritage to control theory, system dynamics is a modelling technique where the whole system is modelled at an abstract level by modelling the sub-systems at component level and aggregating the combined output. This allows us to use feedback/feedforward from one component to another within the system, which unfolds when output is viewed against time. Let's delve into this further by using an example of population dynamics.

Consider a human population growth within a city. Typically, we know from our life experiences that the population grows by people being born and also by people migrating (national and international) into the city. Conversely, the population recedes by people dying and people migrating out of the city. This is our mental model (i.e. view of the world or system around us) and it is useful in modelling process, as it helps to develop the mechanics in our model. We can further the model development by drawing this process graphically and then providing some numbers.



In an unconstrained population system, the birth of new individuals will result in population growth unless it is balanced by the death of individuals within the same system. Also, immigration (migration in) will continue to affect growth unless it is balanced by emigration (migration out). However in reality, population growth is more complex as it depends on many other factors, such as health care, resources, economy and social welfare; all of which are ignored here to allow for a clear explanation of the modelling process, rather than the model itself.

The above system can be written in mathematical form as shown below.

$$Population\ (at\ time, t) = Population\ (at\ time\ t - dt) + Change\ in\ population$$

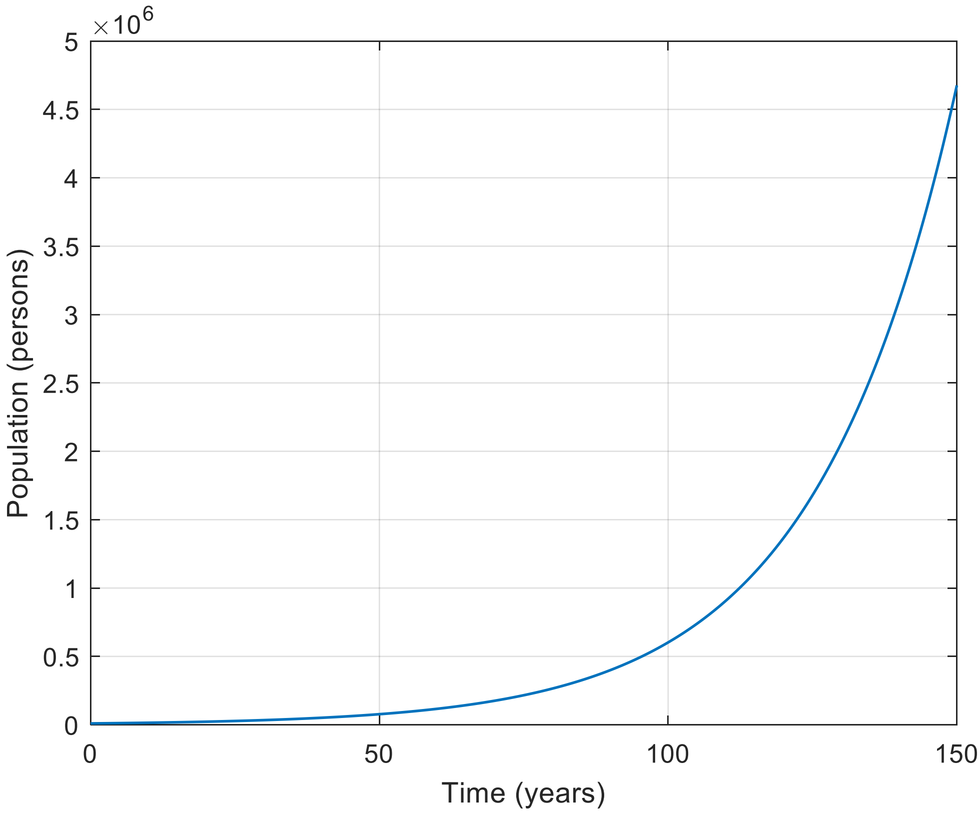
Where dt is a small change in time so, t-dt is basically one time step in the past. Change in population occurs due to addition and removal of people according to birth, death and migration. So, effectively, present value of population $(P + \Delta P)$ is equal to past value of population (P) plus the change in population (ΔP) that occurred during time dt. This can be further elaborated as,

$$\begin{aligned} Population\ (at\ time\ t) \\ &= Population\ (at\ time\ t - dt) \\ &+ dt \\ &\times \{Population\ (at\ time\ t - dt) \times Birth\ rate \\ &+ Population\ (at\ time\ t - dt) \times Immigration\ rate \\ &- Population\ (at\ time\ t - dt) \times Death\ rate \\ &- Population\ (at\ time\ t - dt) \times Emigration\ rate\} \end{aligned}$$

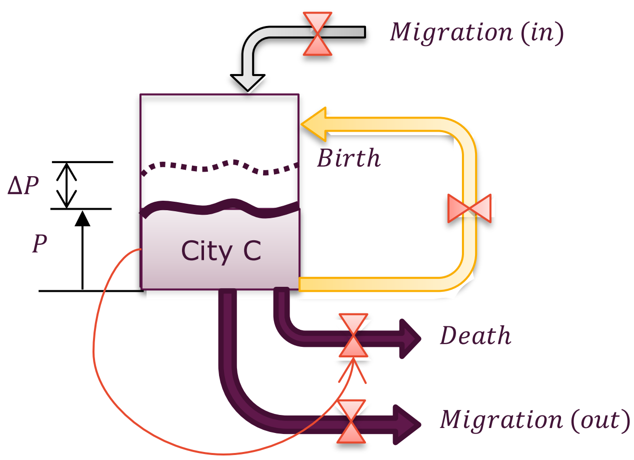
In reality, these rates also change as time progresses, some due to internal factors within the system itself (endogenous) for example, changes in death rate affected by poor sanitation due to increased population size. Rates can also change due to external influences (exogenous) for example, change in death rate due to changes in health care. However, in this example, we shall consider all rates to be constant as a function of time. Let us consider our initial conditions of the system (at present time) as below.

$$Population = 10{,}000\ persons$$
$$Birth\ rate = 45\ persons\ per\ 1000\ persons\ per\ year$$
$$Immigration\ rate = 2\ person\ per\ 1000\ persons\ per\ year$$
$$Death\ rate\ = 5\ persons\ per\ 1000\ person\ per\ year$$
$$Emigration\ rate = 1\ persons\ per\ 1000\ persons\ per\ year$$

With this information and our model structure (i.e. the above mathematical formation), we can simulate our system to predict future population levels up to a time of our choice, for example 10 years. Here, we need to choose dt (simulation time step) to be very small compared to the quickest rates in the system. We shall choose it to be 1 week or 1/52 year so that the simulation yields as close to analytical results as possible. When above equation is simulated from year 0 to year 150, we see the exponential growth of population, as shown in picture below. Remember, birth rate is higher than the death rate! Please note that we have chosen large enough timescale such that changes are clearly visualised. However, the timescale for forecasting future should be chosen appropriately that meets the criteria of goal setting in policy making process.



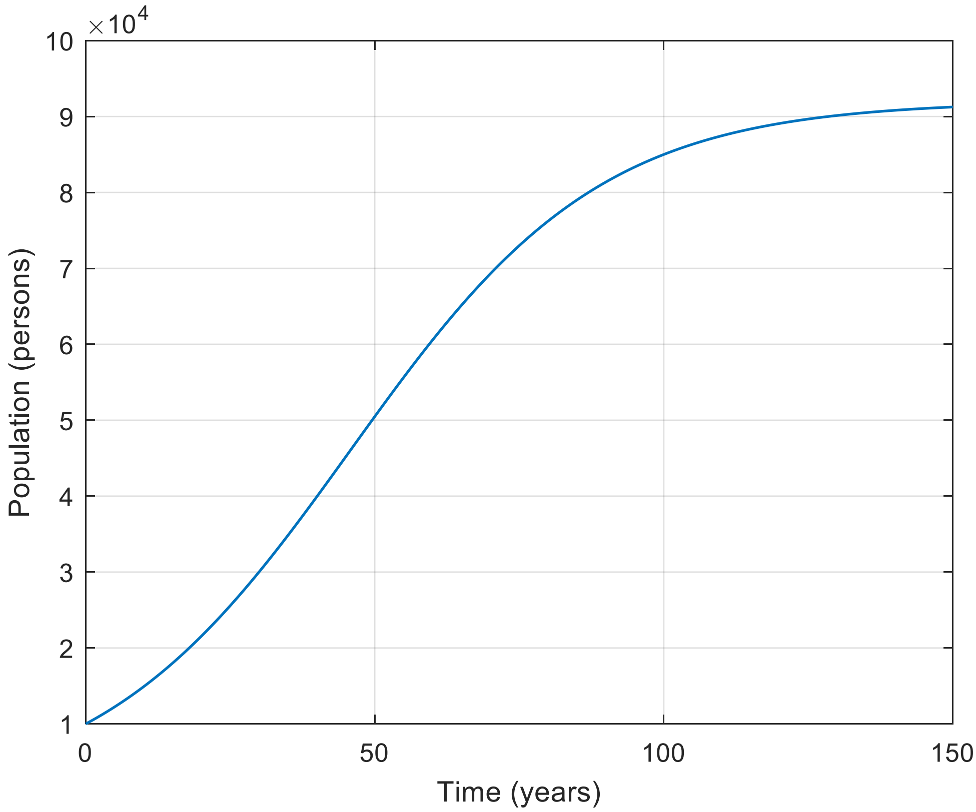
One might say that this is not what we observe in reality. We expect the population to saturate at some point, due to various factors mentioned earlier; therefore let's presume that as the population becomes dense in a constrained environment, individual health deteriorates (due to poor hygiene in a dense environment and lack of resources) and so the death rate increases as the population increases. This leads to another feedback (similar to births and deaths being proportional to number of persons), whereby the density of the population influences the death rate. We can capture this information graphically by updating our previous figure accordingly.



Mathematically, we can make changes to our previous equation as shown below in red.

$$\begin{aligned} &Population\ (at\ time\ t) \\ &\quad = Population\ (at\ time\ t-dt) \\ &\quad + dt \\ &\quad \times \Big\{ Population\ (at\ time\ t-dt) \times Birth\ rate \\ &\quad + Population\ (at\ time\ t-dt) \times Immigration\ rate \\ &\quad - Population\ (at\ time\ t-dt) \times \left(\frac{Population\ density\ (at\ time\ t-dt)}{carrying\ capacity} \right) \times Death\ rate \\ &\quad - Population\ (at\ time\ t-dt) \times Emigration\ rate \Big\} \end{aligned}$$

Here, instantaneous population density is population (at time t - dt) divided by available area. Let's consider area = 50 sq.km and carrying capacity = 200 persons per sq.km. Simulating this again, gives us the following results. Please note that we have assumed that the death rate increases linearly with the population density which is of course simplistic view of the complex interaction of population with the resources.



Here, we can see the logistic growth resulting from this additional feedback. As you can see, we have made a small change to our mathematical formulation but we see quite different results. One can observe that the birth rate is dominant between year 0 and 50 which is why you see the initial upward exponential growth in the population. After year 50 the deaths become dominant, due to the effect of carrying capacity, and so the growth slows down and reaches a value that is balanced by the birth and death rates. It should be stressed here that the output (population in our case) from the system dynamics model is dependent on intial conditions and rates. So, at each simultaion step, the future value depends on the present value and sometimes the past value as well. Therefore, the mathematical structure is generic and is transferrable to other similar system, so long as the system properties are similar.

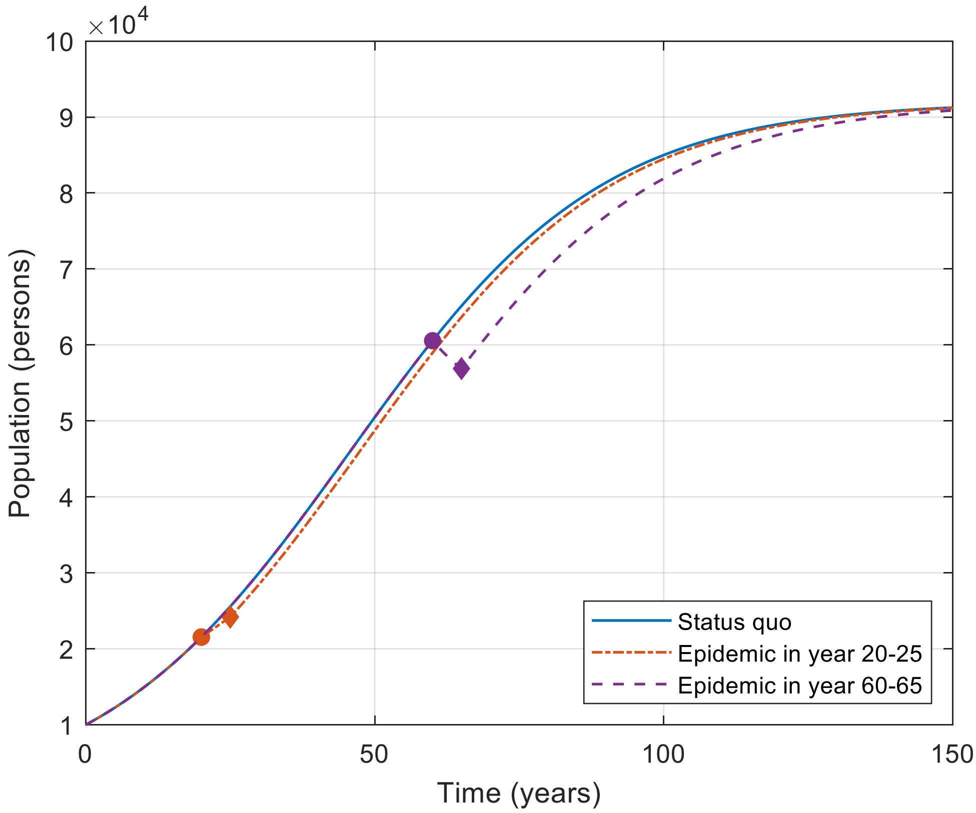
The mathematical formulation presented here is very simple and in a descretised form. This way of simulation is known as Euler's method. Most dynamical systems are formulated using differential equations and presented in state-space form, which is solved using Rung-Kutta method for better efficiency in computation as well as accuracy. One can derive analytical solutions to these models using advanced mathematics. However, implentation of SD models as presented here, does not require one to have experts level knowledge of advanced mathematics.

One can further play with this model and change the values of rates and even make them function of time such as step, ramp or any other to see its effects on the output from the system, i.e. population. This shows flexibility of system dynamics technique to change parameter or part within the system to reflect the changes made in the system.

How is it useful for impact assessment?

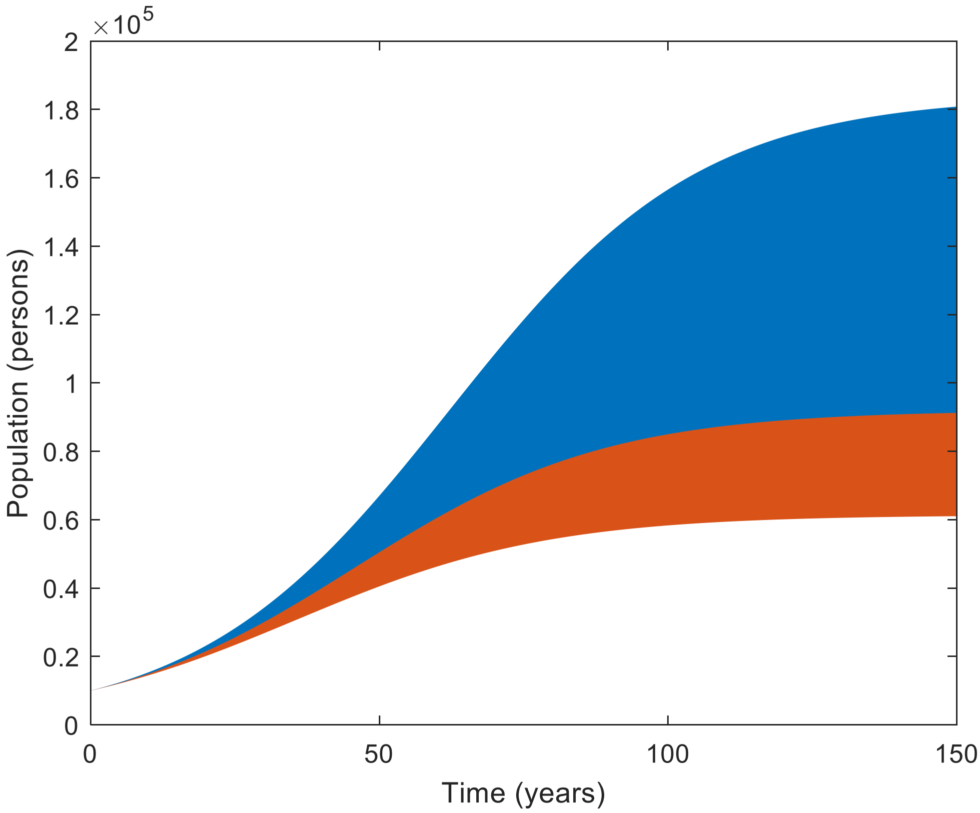
Our society is a very complex system and it is mostly only possible to model different small sub-systems that are relevant to the problem at hand. More specifically, within LEVITATE, we have a transportation system that is undergoing transformation (in terms of introduction of connected and automated transport systems) and this system has complex relationship with the users who are defined by factors such as income, age, education level, etc. Consequently, complex dynamics emerge when all these sub-systems comprising of population dynamics, employment dynamics, housing dynamics, etc., interact with each other. System dynamics framework provides a basis to understand them, as well as interact with the model by playing 'what if' scenarios to look at (a) external disturbances and (b) the effects of policy measures. Furthermore, it is very much a 'white box' modelling approach, that allows you to know which part of the system causes the behaviour you see and how it affects the overall system.

Let us come back to our population dynamics model and say, due to an external event (disturbance) that triggered in year 20, for example an epidemic which doubled the death rate until year 25, what would be its effect on the population in the future? Now, what if the same epidemic occurred in year 60 until year 65? We would simulate this again using the same model by changing the death rates between the periods specified. Compare these scenarios with our baseline without the epidemic, we can see that the effect is much larger when the population is in its developed state, i.e. slowed growth at year 60. So, the same event can yield different results, and the effect very much depends on the state of the system. This can only be realised by simulating the system with its characteristic parameters.



Once a model is obtained which represents the system reasonably well, it can be subjected to different policy scenarios to see the effects of those policies on the overall system. Usually, these models have multi-input multi-output structure and one can further use multi-objective optimisation to deduce optimum policies for a system at hand. To clarify, optimum policies for one city could be different to optimum policies for another city, due to different states of the system and goal objectives.

Often, during the modelling stage, we estimate some parameters used in the model using past data (if available) or sometimes even an educated guess. It becomes important to see how uncertainty in chosen value of parameter being correct, affects the overall system. This is done via sensitivity analysis. One can vary one or many parameters within the model, sequentially or randomly, within certain bounds, to see its effect on the system. The latter (random variation) is known as Monte Carlo sensitivity analysis. In our population dynamics example, we might ask what effect an incorrect estimation of death rate input has on the population estimation. We can simulate this by varying death rate from 2.5 to 7.5 person per 1000 person per year.



We find that it has a nonlinear effect on the estimation, such that relatively large deviation occurs when we vary from 5 towards -50% and relatively small deviation occurs when we vary it from 5 towards +50%. In this case, large deviation in output suggests that this parameter must not be ignored in this model and, also that it must be estimated as closely to reality as possible. This treatment of sensitivity analysis can also be applied when choosing policies and fine-tuning the specific parameters of these policies.

What are its drawbacks?

Since, the system dynamics techniques operate using aggregated parameters of the system and its constituents, it suffers from the loss of heterogeneity at an individual level. This can be somewhat compensated by compartmentalising system by attributes. For example, zone wise, age wise, income wise splitting of population within the city. Interaction within these compartments, in terms of flows from one compartment to another, can be implemented using logit functions or by providing elasticity parameters to compensate for the heterogeneity in the system. However, this approach cannot match the high fidelity given by the agent-based modelling. One has to evaluate the need of heterogeneity and whether or not it is relevant to the problem.

Often the geometrical structure is omitted, as it is not necessary for the problems tackled by this method. In our example, simulation outputs do not change if the city has boundary walls in the form of a square or circle or not at all. All we are concerned of is the population within the defined area. Similarly, for a model that looks at mode choice, it is not a major concern which route one takes to get from A to B, but instead what mode one takes depending on the cost and convenience. This model will suffer slightly due to lack of 'real' journey time and congestion information, which could also influence the choice of mode (such as car or bike). If the latter is more dominant and important, one should implement the problem in an agent-based modelling environment.

Final remarks

System dynamics is a powerful way of modelling a system at an abstract level. Final points on its strengths and usefulness can be summarised as below.

- Feedback within the system can be handled easily and one can see the effect of complex feedback via numerical simulations.
- Interconnected systems can be integrated very well and allows one to extend the model as well.
- Structure determines behaviour – same model, different behaviours due to states of sub-systems/constituents.
- Future values depend on past values.
- Mathematical complexity of large complex system does not hinder modelling, as the system is solved by using solvers using discretised system.
- Allows one to play with 'what if' scenarios easily and faster. It allows one to change the strength and timing of external disturbances as well as of policy measures that might be applied.
- Provides a deeper understanding of the system, as one knows what effects are generated in the system, due to a particular cause presented to it.

In LEVITATE, system dynamics will mainly be used to evaluate the impact of policy interventions (for example, road use pricing or the introduction of last-mile shuttles) during a transition period of increasing AV percentage. The impact indicators will be typically commuting distance, modal split, etc. as a function of time so that the evolution of impacts over the long-term duration can be compared against various scenarios.

Examples of SD method implementation for impact assessments

- MARS model
  - https://www.fvv.tuwien.ac.at/forschung/mars-metropolitan-activity-relocation-simulator/overview/
- ASTRA and ASTRA-EC models
  - http://www.astra-model.eu/astra-ec-home.htm
- COVID-19 models
  - https://exchange.iseesystems.com/public/isee/covid-19-simulator/index.html#page1
  - https://insightmaker.com/insight/189857/Pandemic-Exploring-the-Dynamics-of-a-Novel-Infection